Tricks and Tips for Teaching Arithmetic

Large difficulties with long division and fractions point to missing foundations in arithmetic and numeracy that need to be repaired before further progress in Math can be made. This article gives some practical advice on how to cement your child's foundations in Math.

Practice Makes Perfect

Our son is trying out for Little League in January. His skills in catching, throwing, etc. are not quite there, but they are rapidly improving. How are we doing this? We practice daily. Similarly, your child's brain can be thought of as a muscle that needs a daily work out for it to be stronger and quicker. Just like baseball or any other sport, regular daily practice makes all the difference in the world. If you think it's hard to find the time to practice when your child is in Elementary, you'll soon realize that it will be impossible in Middle and High School. Time commitments grow as your child progresses through school. Make the time now, while they are still in Elementary, to practice Arithmetic by scheduling it into your daily life. Half an hour a day will produce wonders much more quickly than you realize. If you fail to do so while they are in Elementary, I can guarantee that you will regret it when they are in Middle and High School. The SAT requires an automatic knowledge of basic skills that take time to learn.

Start with Numeracy

It's surprising how often this is skipped or not fully realized. Many parents get their children to count up to 30 or 100 and call it quits. They also fail to work on number writing and recognizing numbers out of sequence. Young students for instance often don't see a difference between 12 and 21, or 13 and 31. Often weak numeracy is accompanied by writing numbers in reverse and the like.

From 1 to 30 - and Mix it up

For younger students, start off by teaching them their numbers up to 10, then up to 20, and then up to 30. As their mastery improves, take numbers out of sequence and test them on these. You may be able to practice this as you drive them around town.

From 1 to 1,000 and back again

As your child progresses, push them to master all the numbers between 1 and 1,000. Many older students can vastly improve their numeracy in just 20 days. Find a 10 * 10 empty grid (i.e. 10 lines of 10 blank spaces each, totaling 100) and create 20 copies. The goal is to have your child write 1 to 1,000 and back down again. The first day you assign them 1 to 100, the second day 101 to 200, and so on until you reach 1,000. Having reached 1,000, go back from 1,000 to 1. Thus on the 11th day you will have them write from 1,000 to 901, then 900 to 801, and so on until you reach 1. This simple exercise will correct their number writing skills and cement a knowledge of the numbers between 1 to 1,000. You can then use this knowledge to teach further place values up to 1,000,000 and beyond.

Once Numeracy is Conquered, add Addition

The easiest way to introduce addition is to simply ask: "What number comes next?" For example, 5 → ? You can then formalize this with 5 + 1 = ? Play with the + 1's until they are pretty comfortable or start to get a little bored, and then introduce + 2's, then +3's, and so forth. Obviously they require a reasonable grounding in numeracy to get started, but they need not have fully mastered it. It is a good idea in fact to use addition to review numeracy and link it back to addition. For example, you can ask what is 99+1=? and 999+1=? as ways to cement and review.

Be Careful with Counting on Fingers

Many students are introduced to addition through counting on fingers. This is a double edged sword, as many students become completely dependent and/or uncomfortable in adding without counting. This can slow them down considerably. Yes, you want your child to know how to add by counting. For instance you want them to know that in 3 + 8 that they can start with the larger number 8 and then count 3 up from there. In the end though, you want your child to memorize that 8 + 3 or 3 + 8 is 11, without counting. Committing this to memory will help them immeasurably with Math later on, as it will open the door to more easily learning more complex concepts through a greater fluency with numbers.

Fluency and Memorization are Important

You want your child to memorize every combination of addition for 1 through 10, i.e. any combination of 1 through 10 + any combination of 1 through 10, starting from 1+1 to 10+10. It's also a good idea to introduce the concept of 0 and how to work with that. By having all these combinations memorized, your child will have a much greater fluency with Math throughout their lives. Rote learning is looked down upon in general in education, but in Math memorizing this foundation will open big doors later on. We've seen many older students struggle with Algebra and lose their confidence because they don't have the basics down, and may still be counting to add 8 + 7 in late Elementary and early Middle School.

Let them get it Wrong

Many students are afraid to let go of counting because they've been taught to value accuracy more than fluency. Your child will rarely be reprimanded for slowly counting out a calculation, but they will face consequences if they quickly jump to the wrong answer through their memory. Yet they will face grave consequences in later testing if they are slow - as they will not get the test done in time. Speed tests are a good thing in Math as they push students to trust their memory, and this enables them to move much more quickly and complete their tests. There is often too large a focus on accuracy, and this comes at the expense of fluency. Students need to trust their memory and make mistakes in order to correct their memory and improve.

Use Addition to Master Subtraction

We've tried several different curricula with thousands of children over the years. One famous curriculum focuses on completely mastering numeracy, then addition, then subtraction, and so on. We found this too simplistic and boring for most of our students. It also failed to improve their conceptual understanding. A much better approach is that of teaching "fact families," which is popular in school. Fact families are where students learn that 3+8 and 8+3 are 11, and also that 11-3 is 8 and 11-8 is 3. All they need to do is memorize the three numbers together and they will gain all four calculations. Unfortunately school rarely gives students sufficient practice to memorize these fact families. This is something you can easily do at home, being sure to emphasize the goal of memorizing the numbers together. An additional benefit of the fact family approach is that students gain a more solid conceptual foundation of how addition and subtraction are related, which helps later on with Algebra. Fact families also help students leverage their addition knowledge into subtraction.

From Addition to Multiplication

The best way to teach multiplication is to teach children that multiplication is simply a condensed expression of addition. For example 7 * 5 is really 5 + 5 + 5 + 5 +5 +5 +5 or 7 + 7 +7 +7 +7. The student can take their pick of how to work it out. When a student asks me what 7 * 6 is, I always go back to this and get them to work it out. They can choose to use whichever factor is easier for them to add (either 6 or 7 in this case). Many times they tell me that they cannot do it, and then I simply write it out for them and get them to add it up with me verbally. After we do this a few times, they simply need a little encouragement to do it on their own. They are now free to work out any of the times tables on their own. Teaching the times tables in this way cements their knowledge quickly through addition practice. With every new factor, there is less to learn as they have already memorized every factor up to that one previously. For example when a student is learning their 7 times tables, they should already know all the combinations up to 7*6 and in reality they only need to learn 7*7 through 7*12 (they will already know 7*10).

Weakness in Addition shows up in Multiplication

One thing we've learned from teaching students their times tables is that it is far more difficult to teach students who have weak addition skills. This is because they make many more mistakes when adding 7 + 7 + 7 + 7 + 7 in 7*5 than a student who has good addition skills. So a weakness in addition directly impairs their ability to learn multiplication. The times tables are thus one of the first watersheds that a student does not know their foundations as well as they should, and this is often when we see students come through our doors. Mom will tell us that Johnny has trouble learning their times tables, and the issue is usually much deeper. Johnny can't in fact add and subtract very well, and may not have a solid grasp of numeracy. In such a case we will be sure to patch these foundations in as we also try and teach them their times tables.

Back to Fact Families with Division

Just like subtraction is the mirror of addition, so is division the mirror of multiplication. Fairly early on, it is a good idea to introduce the idea of fact families in multiplication and division. For example 7 * 8 is 56, 8 * 7 is 56, 56/8 is 7, and 56/7 is 8. By memorizing these three numbers together (7, 8, and 56) the student gains a fluency with all four calculations. Similar to fact families in addition and subtraction, this also enhances their conceptual understanding and better prepares them for Algebra.

Wrap it all up in Long Division

Long Division has a certain elegance because it wraps up all the skills of numeracy and arithmetic into one neat little bundle. To perform long division well, at least for a two digit or larger divisor, one needs to be able to make an estimate using rounding (requiring strong numeracy); then multiply this out; then subtract; then bring down; and repeat until completed. Long division is a real watershed as to whether your child is prepared to learn more complex and interesting concepts in Math.

For example in 54,563 divided by 61, you will need to estimate 545/61 - rounding 545 to 55 and 61 to 6, thus 55/6 which is 9. This is the first step. You then write 9 above 545 and multiply 61 * 9 to get 549. You should then subtract 545 - 549 - wait, you can't do this. Since you made a good initial estimate, you only need to reduce your answer from 9 to 8 and recalculate 61 * 8 to get 488. Now you need to calculate 545-488 to get 57, then you bring down the 6 to get 576. Having already worked out 61*9 you can simply write a 9 above 5456 and subtract 576-549 to get 27. Bring down the 3 to get 273. Divide 273 by 61 by estimating 27/6, which is 4. Write 4 above 54,563. Multiply 61*4 to get 244. Then subtract 273-244 to get your remainder of 29. In this way you have solved 54,563/61 and obtained the answer of 894 with a remainder of 29. You should then quickly check this by multiplying 894*61 and then adding 29 to get 54,563. There is no way you are going to be able to calculate long division quickly or well if you have ANY weaknesses in numeracy, addition, subtraction, multiplication, or division. Thus long division is a fantastic inflection point for seeing if your child is truly prepared to learn algebra and the like. With the foundations in place, they will be able to succeed in Math through their lifetime. Without them, they will always struggle and never enjoy or appreciate Math.

Teach your kids numeracy and arithmetic! Schools spend far too little time on these basic skills, which are the underpinnings for everything else.